Magnetic Force Exerted by the Aharonov-Bohm Line

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The problem of the scattering of a charge by the Aharonov-Bohm (AB) flux line is reconsidered in terms of finite width beams. It is shown that despite the left-right symmetry in the AB scattering cross-section, the charge is deflected by the AB-line as if by the "Lorentz" force. The asymmetry originates from almost forward scattering within the angular size of the incident wave. In the paraxial approximation, the real space solution to the scattering problem of a beam is found as well as the scattering S-matrix. The Boltzmann kinetics and the Landau quantization in a random AB array are considered.

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The Aharonov-Bohm (AB) flux line is an idealized construction originally designed to discuss the role of the vector potential in quantum mechanics [1]. The magnetic field around the AB-line is zero but the gauge vector potential is finite being generated by the magnetic flux Φ concentrated in the line. Nowadays, the AB-line is very popular in various contexts: As a carrier of Chern-Simon's field, the AB-line attached to a particle allows one to build two-dimensional objects obeying a fractional statistics (anyons), or composite fermions in the theory of strongly correlated electronic systems (Quantum Hall Effect or high-T$_c$ oxides). In many respects, vortex lines in superfluids are similar to AB-lines: The superflow around a vortex line in $He-II$ acts on a normal excitation like the vector potential of an AB-line on a charge. Problems of the vortex dynamics, e.g the existence of the Iordanskii force [3], are currently under active debate [4–6].

There exists a vast literature on a charge interacting with an AB-line (for a review see [2]). Surprisingly, there is a question which is still controversial. One can formulate the question as: Does the AB-line exert a Lorentz-like magnetic force on a moving charge? Or, in other words, whether the charge is deflected right-left asymmetrically revealing the absence of the mirror symmetry ($\mathcal{P}$) broken by the magnetic flux. Despite the fact that the exact solution to the AB-line scattering problem has been known since the original paper of Aharonov-Bohm [1], there is no common opinion on the subject. Different people give opposite answers saying:

No: Any effect of scattering can be expressed via the differential cross-section $d\sigma/d\varphi$, φ being the scattering angle. The cross-section known from [1],

$$\frac{d\sigma}{d\varphi} = \frac{\lambda\!\!\!^{-}}{2\pi}\,\frac{\sin^2 \pi\tilde{\Phi}}{\sin^2 \frac{\varphi}{2}} \tag{1}$$

is $\varphi \to -\varphi$ symmetric, and, therefore, there is no left-right asymmetry in the outgoing wave ($\tilde{\Phi} = \Phi/\Phi_0$, $\Phi_0 = hc/e$, and $\lambda\!\!\!^{-} = 1/k$ is the particle wave length). The net transverse momentum transfer must be zero together with the Lorentz force. The only way to reveal the broken $\mathcal{P}$ is via the interference of the AB- and potential scattering as e.g. discussed in [7].

Yes: Any effect allowed by symmetry is there. The symmetry of the problem is that of a charge in a magnetic field and the transverse force must be finite. As far as calculations are concerned, the force is given by a divergent integral (over φ) or sum (over partial waves). The divergence can be handled and the result seems to be reasonable: it agrees with the derivation based on the momentum balance [2].

The purpose of this paper is to reconsider the problem. The main idea is that the difficulties and ambiguities are due to the combination of two factors: (i) the infinite range of the vector potential; of the AB-line; and (ii) the infinite extension of the incident plane wave in the Aharonov-Bohm solution, allowing absolute resolution in the direction of propagation. The plane wave solutions are not suitable for extracting physics from them because of the forward scattering singularity. Instead, one should analyze beams with a finite angular size. Then, the singularity at $\varphi = 0$ is regularized in a natural way.

The paper is organized as follows. First, to check the very existence of the transverse "Lorentz force", I analyze the gedanken experiment where an incoming beam of a finite effective width W hits an AB-line and the deflection of the beam measures the magnetic force exerted by the line. The calculations are done in paraxial approximation [8]. At the expense of fine details on the scale of the wave length $\lambda\!\!\!^{-}$, the paraxial approximation allows one to find rather easily the wave function for small scattering angles at distances $\gg \lambda\!\!\!^{-}$. As expected from the symmetry arguments, a finite deflection is observed. However, the deflection angle being $\sim \lambda\!\!\!^{-}/W$ tends to zero when the incoming beam transforms to an infinite plane wave, $W \to \infty$ (in agreement with the argument based on the symmetry of the cross-section). The transverse momentum transfer is found for arbitrary incoming wave, and the result is presented in terms of the effective force. To understand how the effect of many lines adds together, I consider a random array of the AB-lines, a model similar to that of Desbois *et al.* [9]. On average, the anomalous asymmetric small-angle scattering amounts to an effective magnetic field proportional to the density of lines.

The kinetic equation and the resistivity tensor, as well as the Landau quantization are discussed. Implications of the results are discussed at the end of the paper.

Consider the experiment where a particle of mass m, charge e and momentum $p = \hbar k = \hbar/\lambda\!\!\!^{-}$ moves on the $x-y$ plane in the x-direction from $-\infty$ and meets the Aharonov-Bohm line piercing the plane at $\boldsymbol{r} = 0$. The distribution with respect to the transverse coordinate y is measured, and the averaged value $\bar{y}(x)$ defines the "trajectory" from which the deflection of the particle by the AB-line is extracted. To make the transverse coordinate meaningful, the stationary incident wave is beam-like with a finite transverse size $W \gg \lambda\!\!\!^{-}$ (e.g. a wave having passed through an aperture of the width W).

In the paraxial theory [8], a particle moving at a small angle to the $x-$axis is described by the wave function of the form $\Psi = e^{ikx}\psi$, where $\psi(x,y)$ is slow, $|\nabla\psi| \ll k|\psi|$. Neglecting $\frac{\partial^2\psi}{\partial x^2} \ll k\frac{\partial\psi}{\partial x}$ in the stationary Schrödinger equation, one comes to the paraxial equation:

$$iv\partial_x\psi = -\frac{1}{2m}\partial_y^2\psi \tag{2}$$

where the velocity $v = \hbar k/m$ and $\boldsymbol{\partial} \equiv \hbar\boldsymbol{\nabla} - i\frac{e}{c}\boldsymbol{A}$, $\boldsymbol{A}$ being the vector potential chosen below as $A_y = 0$ and $A_x = -\Phi\delta(x)\,\theta(y)$.

The incoming wave, $\psi(x<0,y)$, is controlled by conditions of the experiment such as screens, apertures *etc.* Leaving the preparation of the incoming wave out of the picture, $\psi(x=-0,y) \equiv \psi_{in}(y)$ can be taken as the input to the scattering problem. Solving Eq.(2) in the immediate vicinity of the line $x = 0$, where the vector potential is concentrated in the chosen gauge, one finds

$$\psi(+0,y) = \psi_{in}(y)\exp\big(-2\pi i\tilde{\Phi}\theta(y)\big)\ . \tag{3}$$

Further propagation is free, and the outgoing wave is

$$\psi(x,y) = \int_{-\infty}^{\infty} dy'\,G_0(y-y';x)\psi(+0,y')\ \ ,\ \ x>0, \tag{4}$$

where $G_0(y;x) = \theta(x)\sqrt{\frac{k}{2\pi ix}}e^{\frac{ik}{2x}y^2}$. Or, using Eq.(3),

$$e^{i\pi\tilde{\Phi}}\psi(x,y) = \cos\pi\tilde{\Phi}\ \psi_0(x,y) + i\sin\pi\tilde{\Phi}\int_{-\infty}^{\infty} dy'\,G_0(y-y';x)\frac{y'}{|y'|}\psi_{in}(y')\ ,\ x>0, \tag{5}$$

where $\psi_0(x,y) = \int_{-\infty}^{\infty} dy'\,G_0(y-y';x)\psi_{in}(y')$ is the solution in the absence of the line.

Given the incoming wave $\psi_{in}(y)$, Eqs.(4) or Eq.(5) allows one to find the outgoing wave at $x \gg \lambda\!\!\!^{-}$ in the small angle region $|\varphi| \ll 1$, $\varphi = y/x$.

For the *infinite* plane incident wave, *i.e.* $\psi_{in}(y) = \psi_0(x,y) = 1$, Eq.(5) immediately gives (up to a gauge transformation) the Aharonov-Bohm solution at $|\varphi| \ll 1$ [2], thus confirming the validity of the paraxial approximation. The paraxial solution $\psi(x,y)$ depends on the coordinates x and y only in the combination $s = y/\sqrt{2\lambda\!\!\!^{-}x} = \varphi\sqrt{x/2\lambda\!\!\!^{-}}$. (When s varies, $\psi(s)$ defines a curve on the complex ψ-plane. One can show that the curve is nothing but the well-known Cornu spiral [10], somehow scaled and shifted; the curve length along the spiral is proportional to s.) At $|s| \gg 1$, *i.e.* $|\varphi| \gg \sqrt{\lambda\!\!\!^{-}/x}$, the wave function acquire the asymptotic form usual for a scattering problem; the corresponding scattering amplitude $\propto 1/\varphi$ giving raise to the AB cross-section in Eq.(1) ($\varphi \ll 1$). In the forward direction $y = s = 0$ the second term in the r.h.s. of Eq.(5) vanishes, and, in agreement with [2,11], $|\psi|^2 = \cos^2\pi\tilde{\Phi}$, whatever distance from the AB-line. The anomalous behaviour when the wave function differs from the incident wave and at the same time does not depend on the distance to the scatterer, takes place at $|s| \lesssim 1$ in the progressively narrow angle range $|\varphi| \lesssim \sqrt{\lambda\!\!\!^{-}/x}$. This is the singularity which causes calculational problems in the standard partial wave analysis of the scattering theory. Obviously, the singularity is absent when the direction $\varphi = 0$ is "blurred" as it is the case if the incident beam is of a finite angular size. When regularized, the forward scattering anomaly turns out to be responsible for the asymmetry in the AB-scattering as it is shown below.

A *finite* width beam $\psi_{in}(y) = \exp(-|y|/W)$, generates at $x \gg W^2/\lambda\!\!\!^{-}$ the following angular distribution, $P(\varphi)$, in the outgoing wave ($P(\varphi)d\varphi = x|\psi|^2d\varphi$):

$$P(\varphi) = \frac{2\lambda\!\!\!^{-}}{\pi}\frac{(\varphi\sin\pi\tilde{\Phi} - \varphi_0\cos\pi\tilde{\Phi})^2}{(\varphi^2+\varphi_0^2)^2}\ \ ,\ \ x \gg W^2/\lambda\!\!\!^{-}\ , \tag{6}$$

$\varphi_0 = \lambda\!\!\!^{-}/W \ll 1$ being the beam angular width. As expected, the angular distribution is regular at $\varphi = 0$. Importantly, if $\varphi_0 \neq 0$, the distribution is *asymmetric* at the angles, $|\varphi| \lesssim \varphi_0$. To quantify the asymmetry in general case, I calculate below the integral effect *i.e.* the deflection of the beam as a whole ("the trajectory bending").

The transverse position of the particle at a given x is defined as $\bar{y} = \int_{-\infty}^{\infty} dy\,y|\psi(x,y)|^2$, and the angle of propagation is $\bar{\varphi} = d\bar{y}/dx$. It follows from Eq.(2) (analogous to the Ehrenfest theorem) that $d\bar{y}/dx = \langle\hat{p}_y\rangle/mv$, $\hat{p}_y$ being the kinematical momentum. In the chosen gauge, $\langle\hat{p}_y\rangle_{out} = \int_{-\infty}^{\infty} dy\,\psi^*(x,y)\frac{\hbar}{i}\frac{\partial}{\partial y}\psi(x,y)$ with $\psi(x,y)$ given by Eq.(4). In the force free region, $\langle\hat{p}_y\rangle_{out}$ does not depend on x, and the integral can be conveniently calculated at $x \to +0$ using Eq.(3). The deflection angle $\Delta\varphi \equiv \bar{\varphi}_{out} - \bar{\varphi}_{in}$ is $\Delta\varphi = \Delta p_y/p$ where $\Delta p_y = \langle\hat{p_y}\rangle_{out} - \langle\hat{p}_y\rangle_{in}$, After simple calculation [12],

$$\Delta p_y = -\hbar|\psi_{\rm in}(0)|_N^2\sin 2\pi\tilde{\Phi}\ , \tag{7}$$

where $|\psi_{\rm in}(0)|_N^2 = |\psi_{\rm in}(0)|^2/\Big(\int_{-\infty}^{\infty} dy|\psi_{in}|^2\Big)$.

We see that indeed the AB-line deflects particles asymmetrically, with the left-right asymmetry $\Delta\varphi = \Delta p_y/p$ controlled by parity-odd Φ. One could naively expect the asymmetry thinking in terms of the classical Lorentz force exerted by the magnetic field "inside" the line. However, in contrast to the classical expectations, $\Delta\varphi$ is *not* proportional to the field strength but is periodic in

Φ, revealing a quantum origin of the effect. In agreement with general arguments [2], the deflection Eq.(7) is finite only if $\psi_{in}(0) \neq 0$, *i.e.* the incoming wave must overlap with the line.

From the estimate $W|\psi_{\rm in}(0)|_N^2 \sim 1$ and Eq.(7), deflection $\Delta\varphi$ is of order and never exceeds the beam angular size $\varphi_0 \sim \lambda\!\!\!^{-}/W$, supporting the conclusion made from Eq.(6). This means that the transverse effect cannot be discussed in terms of the scattering amplitude: Within the cone $|\varphi| \sim \varphi_0$, the scattered and incoming wave inevitably overlap and the split of the wave into incoming and scattered components is rather arbitrary. Besides, the forward scattering is singular and, as was discussed by Berry *et al.* [11] one should deal with the full wave function, which is regular in the forward direction, rather than split it into two singular parts.

Instead, one may use the description in terms of the $S-$matrix: $F_{out}(\varphi) = \int_{-\infty}^{\infty} d\varphi' S(\varphi,\varphi') F_{in}(\varphi')$, with the amplitudes F's defined in the momentum representation,

$$\psi = \int_{-\infty}^{\infty} d\varphi F_{in/out}(\varphi) e^{ik\varphi y - i\frac{1}{2}kx\varphi^2}, \, x<0/x>0. \quad (8)$$

It follows from Eq.(3), that $S(\varphi,\varphi')$ is the Fourier transform of

$$S(y,y') = \delta(y-y')\exp(-2\pi i\tilde{\Phi}\theta(y)) \cdot \exp(\delta|y'|)$$

where the last factor with a regularization parameter $\delta = +0$ is introduced with the understanding that the incident wave has a finite extension in the y-direction. Finally, S-matrix reads

$$S(\varphi,\varphi') = \delta(\varphi-\varphi') + \frac{1}{\pi}\sin\pi\tilde{\Phi}\frac{e^{-i\pi\tilde{\Phi}}}{\varphi'-\varphi+i\delta} \quad , \quad \delta = +0 \, . \quad (9)$$

$S(\varphi,\varphi')$ is unitary, its overall phase is gauge-dependent.

For an incident wave with physically reasonable $F_{in}(\varphi)$, this regularized expression for the $S-$matrix gives the outgoing wave $F_{out}(\varphi) = \hat{S}F_{in}(\varphi)$ finite and smooth at any angle:

$$e^{i\pi\tilde{\Phi}}\, F_{out}(\varphi) = \cos\pi\tilde{\Phi}\, F_{in}(\varphi) + \frac{1}{\pi}\sin\pi\tilde{\Phi} \;⨍ d\varphi' \,\frac{F_{in}(\varphi')}{\varphi'-\varphi}\, , \quad (10)$$

where $⨍$ denotes the principal value of the integral. Coming from the $\delta-$function contributions in the $S-$matrix, the first term in the r.h.s. gives a modified incident wave, attenuated and with a phase shift (gauge dependent). The second term reproduces the AB-scattering in the limit of small scattering angles.

It is clear now that the left-right asymmetry in the outgoing intensity $|F_{out}|^2$ may come only from the interference of the two terms in Eq.(10): The cross product is the only contribution to $|F_{out}|^2$ with the magnetic symmetry, *i.e.* odd relative to $\Phi \to -\Phi$. Obviously, the magnetic interference piece is present only at the angles where $F_{in}(\varphi) \neq 0$ *i.e.* of order of the angular width of the incident wave $|\varphi| \sim \lambda\!\!\!^{-}/W$. Loosely speaking, the asymmetry and the magnetic force exerted by the AB-line originates in the interference of the scattered and incident waves.

Qualitatively similar physics was conjectured in Ref. [13]. Importance of "auto-interference" was emphasized in Ref. [14] in the context of the time-dependent problem of scattering of a wave packet by the Aharonov-Bohm line.

The AB-scattering is an interesting example when the forward scattering singularity is not just a nuisance, as in the Coulomb scattering case, but it is totally responsible for a qualitative effect – the asymmetry in the scattering.

In Eq.(7), Δp_y is the transverse momentum transfer per collision. Multiplying it by the collision rate $\dot{N}$, one gets a combination, $\mathcal{F}_y = \Delta p_y \dot{N}$, which has the meaning of the force acting on the charge from the line. The collision rate is found as $\dot{N} = \int_{-\infty}^{\infty} dy j_x(x,y)$, j_x being the current density. In the paraxial approximation $j_x = v|\psi|^2$, and using Eq.(7) the force $\mathcal{F}_y = -\hbar v|\psi(0)|^2$. In terms of the full wave function $\Psi_{in}(x,y) = e^{ikx}\psi_{in}(x,y)$ the effective "Lorentz force" reads in the vector form as

$$\boldsymbol{\mathcal{F}}_L = \hbar\sin 2\pi\tilde{\Phi}\;\,(\boldsymbol{J}_{in}\times\boldsymbol{e}_z) \quad (11)$$

where $\boldsymbol{J}_{in}$ is the current density in the *incident wave* at the position of the line: $\boldsymbol{J}_{in} = \frac{\hbar}{m}\;\Psi_{in}^*\boldsymbol{\nabla}\Psi_{in}|_{\boldsymbol{r}=0}$.

Eq.(11) is in agreement with earlier results [15,13,6] where the force was extracted from (divergent) sums in the partial wave analysis. In passing, if one uses wave packets built from the exact AB-solutions, the sums become converging and the doubts in [5] concerning results of [6,15,13] can be readily rejected.

The deflection in Eq.(7) or the force in Eq.(11) are dependent on the details of the wave-packet. A more simple picture emerges when the charge sees many lines and the structure details tend to average out. I consider a model of an AB-array where the position of a line and its flux are random; the density of the array is d_{AB}. After averaging with respect to the randomness in the array, one comes to the Boltzmann-type equation for the distribution function $n_\varphi(p,\boldsymbol{r})$; $p = |\boldsymbol{p}|$ and φ shows the orientations of the particle momentum $\boldsymbol{p}$. The Boltzmann equation, the derivation [17] of which will be presented elsewhere, reads

$$\boldsymbol{v\cdot\nabla}n_\varphi - \frac{e\tilde{B}}{mc}\frac{\partial n_\varphi}{\partial\varphi} + \frac{1}{2\tau_{_{AB}}}\int_0^{2\pi}\frac{d\varphi'}{2\pi}\frac{(n_\varphi - n_{\varphi'})}{\sin^2\frac{\varphi-\varphi'}{2}} = 0 \quad (12)$$

where

$$\frac{1}{\tau_{_{AB}}} = \frac{2\hbar}{m}d_{\rm AB}\big\langle\sin^2\pi\tilde{\Phi}\big\rangle \, , \; \tilde{B} = d_{\rm AB}\frac{\Phi_0}{2\pi}\big\langle\sin 2\pi\tilde{\Phi}\big\rangle, \quad (13)$$

here $< \ldots >$ denotes the averaging over the probability distribution of the flux of a line.

The structure of the kinetic equation Eq.(12) is rather obvious. The collision integral part corresponds to the picture where the AB-lines play the role of impurities independently scattering particles in accordance with the cross-section in Eq.(1). The divergence of the scattering rate at $\phi \to 0$ is unimportant for kinetics as long as the *transport* scattering time ($= \tau_{AB}$) is finite. Accounting for the left-right asymmetry in the forward scattering, $\tilde{B}$ in Eq.(12) enters like a magnetic field. The bending of the trajectories and the expression for $\tilde{B}$ Eq.(13) can be understood from Eq.(7) or Eq.(11) if one sums up the transverse momentum acquired in sequential multiple collisions.

If typically $\Phi \ll \Phi_0$, then $\tilde{B} \approx B_z$, where $B_z = d_{AB}\langle\Phi\rangle$ is the macroscopic magnetic flux density. In this limit, the AB-array is equivalent to Gaussian $\delta-$correlated random magnetic field. If $\Phi \sim \Phi_0$, the behaviour is essentially non-Gaussian: Unlike induction B_z, the effective field is a periodic function of the flux. As expected, integer and half-integer [7,13] fluxes do not contribute to $\tilde{B}$. For a general distribution in Φ, $\tilde{B}$ and B_z may be in any relation. Remarkably, $\tilde{B}$ and the Lorentz force may be finite even if macroscopic induction $B_z = 0$ and, $\mathcal{P}$ and time-reversal symmetries are macroscopically preserved.

Seeing that Eq.(12) has the standard form and $1/\tau_{AB}$ and $\tilde{B}$ are energy independent, one can immediately write down the (Drude) resistivity tensor: $\rho_{xx}^{-1} = e^2 N_0 \tau_{AB}/m$, N_0 being the particle density, and $\rho_{yx} = (\Omega_c \tau_{AB})\rho_{xx}$ where $\Omega_c = e\tilde{B}/mc$ (see also [13,9]). For the Hall angle θ_H, one gets $\tan\theta_H = \Omega_c \tau_{AB} =< \sin 2\pi\tilde{\Phi} > /2 < \sin^2 \pi\tilde{\Phi} >$. In particular,

$$\tan\theta_H = \cot\pi\tilde{\Phi} \tag{14}$$

if the lines have same flux $\tilde{\Phi}$.

The large θ_H when $\tilde{\Phi} \ll 1$, indicates that the particles move along the Larmor circles. The Landau-type quantization of the periodic motion is then expected. In the semiclassical approximation, the oscillations in the density of states $\delta\rho_{osc}(E) \propto e^{-\gamma}\cos(2\pi E/\hbar\Omega_c)$ in a random Gaussian magnetic field have been considered by Aronov *et al.* [18] (see also Desbois *et al.* [9]). Adjusting [18], the damping parameter reads

$$\gamma = \frac{2\pi}{\Omega_c \tau_{AB}} \frac{E}{\hbar\Omega_c} = \frac{\pi}{2\lambda^2 d_{AB}} \quad . \tag{15}$$

Note that in the semiclassical situation, $E \gg \hbar\Omega_c$, the damping is strong even when $\Omega_c\tau_{AB} > 1$ and the periodic Larmor orbiting is well pronounced. Specific to the random magnetic field environment, the damping Eq.(15) increases with the energy E: Similar to inhomogeneous broadening mechanism, the damping here is due to the fluctuations in the number of the AB- lines threading the Larmor circles, the area of which increases $\propto E$.

In conclusion, I have reconsidered the problem of the scattering by the gauge vector field of the Aharonov-Bohm line. It has been demonstrated that the difficulties in the previous calculations, ultimately related to the infinite range of the vector potential, can be avoided if one deals with a beam of a finite transverse size W rather than a infinitely extended plane wave. As allowed by the low symmetry of the problem, the moving charge is asymmetrically deflected by the line (provided the incident wave overlaps with it) despite the fact that the Aharonov-Bohm scattering-cross section is symmetric. The asymmetry in the angular distribution may be attributed to the interference of the incident and scattered waves, and it exists only at the scattering angles of order of the angular width of the incident wave $\varphi_0 \sim \lambda/W$. The derivation is done in the framework of the paraxial approximation which proves to be very efficient. The magnetic "Lorentz force", i.e. the transverse momentum transfer, has been calculated. Earlier results in [15,16,13,6] are confirmed. For an arbitrary incoming wave, the "Lorentz" force Eq.(11) acting on the charge is expressed via the current density in the incident wave at the position of the line.

The AB-problem solves also the problem of the scattering of a phonon by the superflow around vortex line in $He - II$. The superflow is not a gauge-field, and the equivalence holds only in the lowest approximation with respect to the vortex circulation κ. The "Lorentz force" $\propto \Phi$ acting on the charge translates [16] as the (minus) force $\propto \kappa$ acting on the vortex line, the Iordanskii force [3]. The very existence of the Iordanskii force has been recently doubted on the grounds of hardly verifiable general arguments [4] as well as claiming [5] a technical mistake in the previous works on the subject. The present calculations, which are free from divergences and ambiguities of some earlier papers, support the existing understanding on the Iordanskii force in Helium-II [3,16,6] and its role in the vortex dynamics.

Finally, a random array of AB-lines has been considered. The resistivity tensor as well as Landau quantization in the array has been discussed. The array with typical flux in a line $\sim \Phi_0$ gives an example of random magnetic field system with non-Gaussian fluctuations. An interesting observation here is that the effective Lorentz force seen by a charged particle in the array as well as the Landau quantization may persist even when the macroscopic flux density is zero.

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[1] Y. Aharonov, D. Bohm, Phys. Rev. **115**, 485 (1959).

[2] S. Olariu, I. I. Popescu, Rev. Mod. Phys. **57** , 339 (1985).

[3] S.V. Iordanskii, Zh. Eksp. Teor. Fiz. **49**, 225 (1965) [Sov.Phys.-JETP **22**, 160 (1966)].

[4] P. Ao, D.J. Thouless, Phys. Rev. Lett. **70**, 2158 (1993); D.J. Thouless *et al.*, *ibid*, **76**, 3758 (1996); C. Wexler, *ibid* **79**, 1321 (1997).

[5] D. J. Thouless *et al.*, *The 9th International Conference on Recent Progress in Many-Body Theories, 21 - 25 July 1997, Sydney, Australia* (cond-mat/9709127).

[6] E. B. Sonin, Phys. Rev. **B 55**, 485 (1997).

[7] J. March-Russel, F. Wilczek, Phys. Rev. Lett. **61** , 2066 (1988).

[8] M.A. Leontovich, and V.A. Fock, Zh. Eksp. Teor. Fiz. **16**, 557 (1946); M. Lax *et al.* Phys.Rev. **A11**, 1365 (1975).

[9] J. Desbois *et al.*, Nucl. Phys., **B 453** [FS], 759 (1995); *ibid* **B 500** [FS], 486 (1997).

[10] M. Born, E. Wolf, Principles of Optics, (Pergamon Press), 1959, p.431.

[11] M. V. Berry, *et al.*, 1980, Eur. J. Phys. **1**, 154 (1980)

[12] Because of the discontinuity of $\psi(+0,y)$ in Eq.(3) at $y=0$, which is an artifact of the paraxial approximation, some caution is needed. The correct limiting procedure, where details of the behaviour at $y=0$ are unimportant, is the following: $\partial\psi/\partial y$ is substituted for $\left(\psi(y+a,x)-\psi(y-a,x)\right)/2a$ and the limit $a\to 0$ is taken *after* the y-integration at $x=+0$.

[13] M. Nielsen, P. Hedegård, Phys. Rev. **B51**, 7679 (1995).

[14] D. Stelitano, Phys. Rev. **D 51** , 5876 (1995).

[15] R. M. Cleary, Phys. Rev. **175**, 587 (1968).

[16] E.B. Sonin, Zh. Eksp. Teor. Fiz. **69**, 921 (1975) [Sov.Phys.-JETP **42**, 469 (1976)].

[17] A nontrivial part of the derivation is to handle the singular small angle scattering which must be treated unperturbatively. For this, the paraxial two-particle disorder averaged Green function is calculated using the path integral representation to Eq.(2). The Boltzmann equation provides same small-scattering-angle evolution as the exact solution given by the two-particle Green function. The classical Boltzmann equation is applicable (within the localization length) in a diluted array when the energy $E\gg\hbar/\tau_{AB}$, *i.e.* $\lambda\!\!\!^{-2}d_{\rm AB}\left\langle\sin^2\pi\tilde\Phi\right\rangle\ll 1$.

[18] A.G. Aronov *et al.* Europhys. Lett., **29** 239 (1995); Phys. Rev. **B 52**, 4708 (1995).